D4.1 First Report on techniques for plant-wide reactive scheduling

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Modelling frameworks, solution approaches and validation

Project Details

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THE COPro PROJECT

The goal of CoPro is to develop and to demonstrate methods and tools for process monitoring and optimal dynamic planning, scheduling and control of plants, industrial sites and clusters under dynamic market conditions. CoPro pays special attention to the role of operators and managers in plant-wide control solutions and to the deployment of advanced solutions in industrial sites with a heterogeneous IT environment. As the effort required for the development and maintenance of accurate plant models is the bottleneck for the development and long-term operation of advanced control and scheduling solutions, CoPro will develop methods for efficient modelling and for model quality monitoring and model adaption.

The CoPro Consortium

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Abstract
This deliverable presents first developments related to the activities of Task 4.1. “Plant-wide rolling horizon reactive scheduling”. Several new modelling strategies have been investigated for the scheduling of multi-product, multi-purpose and multi-stage production facilities as typically met in real industrial plants, similar to the use cases of FRINSA and P&G. First, various mixed-integer linear programming models (MILP) are presented for the short-term scheduling of multi-stage mixed batch and continuous processes. The most promising model is clearly highlighted and an efficient solution approach for large-scale problems is also proposed. The applicability and efficiency of this model and solution strategy is illustrated in a real case study offered by our FRINSA partner. Then, a new model is also proposed for the planning of maintenance activities, a problem of significant industrial interest. The efficiency of the formulation is illustrated in a real case study offered by our P&G partner. The results of this deliverable indicate clearly that the proposed modelling frameworks can: (i) be used for large-scale mixed batch and continuous scheduling problems of arbitrary complexity and (ii) provide the basis for considering uncertainty issues (e.g. in product demand) in complex large-scale problems of industrial interest, towards the development of reactive scheduling approaches.
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1 Introduction

Due to the highly competitive environment in which manufacturing industries are operating, it is critical to maximize productivity and return from scarce resources. Manufacturers must choose between various alternatives to optimally utilize their resources in order to satisfy the customer demand. This decision-making process is described by production planning, scheduling and supply-chain optimization. Production scheduling is especially important in the process industries (pharmaceuticals, chemicals, food etc.). Since manufacturers strive to increase their productivity, while reducing their operating costs, determining the exact allocation, timing and sequencing of jobs in a complex production becomes increasingly challenging.

Acknowledging the importance of efficient production scheduling, academia proposed numerous strategies to deal with the scheduling problem. Over the last 30 years, many diverse approaches, mixed-integer programming (MIP) formulations and algorithms have been proposed. A number of studies addressing the production scheduling problem of these processes can be found across different scientific communities. In the early 1990s attempts have concentrated on general representations and algorithms that cover a wide range of production scheduling problems [1], [2]. Later research focused on exploiting the structure of specific types of scheduling problems. In [3] and [4] the scheduling of multiproduct multistage batch plants using slot-based formulations have been proposed. The important contributions by [5] and [6] are characteristic examples of sequence-based formulations, while [7] and [8] have also developed algorithms that exploit the structure of the problem. A thorough review that covers the short-term process scheduling methodologies can be found in [9].

The importance of applying scheduling solutions on industrial cases has been widely recognized by the industry [10]. However, few attempts have been made that consider production scheduling problems of real-life plants in order to reduce the gap between theory and industrial practice. Therefore, there is a growing interest in the theoretical implementation to industrial practice. One of the main difficulties of this implementation is that scheduling problems are usually NP-hard and problems with large number of resources and tasks can become intractable. Therefore, to obtain optimal solutions, efficient MILP models and solution algorithms are required, considering plant-specific operational constraints. In [11] a rolling horizon approach is utilized to solve a multistage production of medium size, while a real pharmaceutical industry was examined in [12] using an MIP-based decomposition strategy. [13] proposed a solution strategy for make-and-pack processes, which was validated using data of a real consumer goods company.

Most production plants in the food processing industry sector combine both batch processes and continuous operations, thus working in a semi-continuous production mode. This mode is commonly used because production becomes more flexible and equipment can be more efficiently utilized, allowing the production of high-value, low-volume products. Despite the large number of papers in the open literature, most of the contributions focus on either batch or continuous processes. So, very limited work has been reported for mixed batch and continuous facilities (often called semi-continuous processes), despite being very common in the food, chemical, pharmaceutical and other industrial sectors. Furthermore, the application of scheduling techniques and methodologies is mainly limited to medium-size problems which often ignore the complexity of real-life industrial problems. [14] developed a hybrid genetic algorithm, for the lot-sizing and scheduling problem of a
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dairy industry considering perishability issues and [15] studied a real-life yoghurt production process and proposed a novel mixed discrete/continuous time representation.

Within the scope of this work, multiple solution strategies are presented to address the challenging production scheduling of real-life multiproduct multistage process industries. While the focus is on food processes the proposed models are generic enough to be employed to other process industries such as chemicals and pharmaceuticals. A large-scale complex canned fish industry, offered by our FRINSA partner is considered as the basis for the developed modelling frameworks. The plant is comprised of mixed batch and continuous production operations, thus working on a semi-continuous production mode. This work constitutes one of the first systematic attempts to solve the scheduling problem of a food industry of this size.

2 Description of FRINSA plant

In this project the plant of FRINSA del Noroeste located in Ribeira, Spain is considered as the basis for the proposed modelling and optimization scheduling frameworks. The plant produces a large number of canned fish products making it one of the largest producers in Europe. In particular, the plant can produce more than 400 final products and has a production capacity of 3-4 million cans per day. Several stages, mixed batch and continuous, are required for the production of each can (figure 2.1). Initially, the fish stored in blocks is unfrozen and then filled in cans along with the ingredients (e.g. oil, tomatoes, etc.) required by the recipe. The next stage is the sterilization which guarantees the microbiological quality of the product. Finally, the sterilized cans are packaged (labelling, packing into cartons, boxes etc.) into the final products.

The plant is a multiproduct multistage facility combining both batch (unfreezing, sterilizing) and continuous (sealing and filling, packaging) processes with multiple parallel units. An important characteristic of the plant is that all chambers (unfreezing-sterilization) are identical, while each sealing and filling and packing line consists of a different set of machines. In general, the large production demand and high production flexibility (multiple units can process every single final product) increases significantly the complexity. According to the plant operators, the sterilization and packing stages are the most intensive processes of the plant and they constitute the main production bottlenecks. The short-term scheduling horizon of interest is one week (5 days), whereas the processing units are available 24 hours. Sequence-dependent changeovers and inventory limitations have to be considered. The salient characteristics of the scheduling problem is to take into account a large number of design and operating constraints while ensuring demand satisfaction. The overall production schedule is affected by rather conflicting goals such as the optimal use of resources, the minimization of makespan and the reduction of costs and optimal energy use. The project aims to
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increase the plant’s productivity and minimize the total costs of the production (operating, energy, changeover costs etc.).

3 Modelling approach A: Single stage continuous production

In this section a first approach to solve the FRINSA production scheduling problem is presented. Initially all the assumptions and considerations of the approach are defined (section 3.1). Then, the mathematical model is thoroughly analyzed in section 3.2 and finally validation results based on realistic data provided by FRINSA are presented (section 3.3).

3.1 Problem statement

In this approach the optimization is focused on a single production stage, in particular the packing of the final products. The model is based on the assumption that the main bottleneck of the production is the packing stage, therefore no other stage is modelled in detail.

The problem addressed here is formally defined in terms of the following items:

- A known horizon divided into a set of production periods \( n \in \mathbb{N} \) with available production time \( \text{hor}_{j,n} \).
- A set of parallel packing lines \( j \in J \) with given setup time \( \text{su}_{j,n} \).
- A set of sterilization chambers \( st \in ST \) with production capacity \( s^{\text{cap}}_{p} \) for each product.
- A set of products \( p \in P \) with production targets of different due dates \( \text{dem}_{p,n} \), packing rates \( \text{rate}_{p,j} \), inventory limitations \( s^{\min}_{p} \), \( s^{\max}_{p} \) and maximum production capacity for every packing line \( b_{p,j,n} \). \( P_{j} \) are the products that can be processed in each packing line \( j \).
- A changeover operation of time \( \text{sd}_{p,p',j} \) is necessary in each packing line whenever the production is changed between two products.

We assume a non-preemptive operation, no fluctuations in orders (demand uncertainty) and that there are no limitations of resources (manpower, steam, water, etc.). The key decision variables are:

- the allocation of products to packing lines at every period \( n \), \( Y_{p,j,n} \)
- the sequencing of product-lots in every packing line, \( X_{p,p',j,n} \)
- the completion time of the production of every product, \( C_{p,j,n} \)
- the amount of product \( p \) produced in unit \( j \) at period \( n \), \( Q_{p,j,n} \)
- the inventory level of product \( p \) at the end of every production period \( n \), \( S_{p,n} \)

A holistic approach is followed, meaning that all decisions are made by a single MIP-model, so that an objective function representing the total production time is optimized.
3.2 Modelling approach

3.2.1 Conceptual details of the formulation

The proposed model integrates two different modelling approaches. A discrete-time approach is used for mass balance and inventory calculations (production planning decisions), while a continuous-time representation is employed for the unit allocation, timing and sequencing of product lots (production scheduling decisions) as originally proposed in our previous work \cite{15, 16}. This cross-bread of discrete and continuous time representation can be visualized in Figure 3.1.

![Figure 3.1: Mixed discrete-continuous time representation](image)

For the production planning subproblem a fixed time grid is adopted. In particular the planning horizon is divided into $n \in \mathbb{N}$ production periods (days), with period $n$ starting at time point $n-1$ and finishing at time point $n$. Note that the use of a discrete-time approach at the planning level (big-bucket) enables the correct calculation of inventory costs and mass balances at the end of each production period. Material balances for each product are expressed at the end of each planning period in terms of total production $Q_{p,j,n}$, inventory level $S_{p,n}$, and demand $dem_{p,n}$. It is assumed that the produced amounts during a production period become available at the end of the respective period. The production planning and scheduling subproblems are interconnected through the amount $Q_{p,j,n}$ of product $p$ produced in packing line $j$ during period $n$. Variable $Q_{p,j,n}$ is used by the production planning problem for the calculation of variables $S_{p,n}$ for the material balances, while at the same time variables $Q_{p,j,n}$ are subject to detailed sequencing and capacity constraints of the scheduling subproblem. Moreover, the capacity of the sterilizers is taken into account, so that the total produced amount of product $p$ in period $n$, $Q_{p,j,n}$ does not exceed the production capabilities of the sterilization stage.
In the scheduling subproblem, the products are allocated on packing lines using an immediate precedence approach for sequencing [17]. In particular, the assignment binary variable $Y_{p,j,n}$ is introduced to denote the assignment of product $p$ in packing line $j$ during period $n$ and the sequencing binary variable $X_{p,p',j,n}$ is used to denote an immediate precedence between products $p$, $p'$ in unit $j$ within period $n$. This allows us to correctly take into account the changeover times between products. For the timing of processing of product $p$ in unit $j$ during period $n$, variable $C_{p,j,n}$ is introduced.

### 3.2.2 MILP-model

In this section, we present a MIP formulation for the production planning and scheduling of the parallel-unit (single-stage) facility described in section 3.1. Constraints are grouped according to the type of decision (e.g., assignment, timing, sequencing, etc.).

#### Product-lot sizing constraints

Constraints (3.1) set the upper bound of the produced amount of product $p$ in packaging unit $j$ in time period $n$. Binary variable $Y_{p,j,n}$ defines whether a product $p$ is packaged in unit $j$ in time period $n$.

$$Q_{p,j,n} \leq b_{p,j,n}^{\max} \cdot Y_{p,j,n} \quad \forall p \in P_j, j, n$$  \hspace{1cm} (3.1)

#### Product processing time definition

The required processing time $T_{p,j,n}$ of product-lot $p$ in unit $j$ in time period $n$ is calculated through the given packing rate $rate_{p,j}$ of the product in the respective packing line and the produced amount $Q_{p,j,n}$.

$$T_{p,j,n} = \frac{Q_{p,j,n}}{rate_{p,j}} \quad \forall p \in P_j, j, n$$  \hspace{1cm} (3.2)

#### Unit allocation constraints

Constraint set (3.3) ensures that a packing line $j$ is active in period $n$, when at least on product $p$ is assigned to it in this production period ($Y_{p,j,n} = 1$). In addition, constraints (3.4) state that in case no product $p$ is assigned to line $j$ at period $n$, then the unit allocation variable $V_{j,n}$ needs to be zero.

$$V_{j,n} \geq Y_{p,j,n} \quad \forall p \in P_j, j, n$$  \hspace{1cm} (3.3)

$$V_{j,n} \leq \sum_{p \in P_j} Y_{p,j,n} \quad \forall j, n$$  \hspace{1cm} (3.4)
Timing and sequencing constraints

Constraints (3.5) and (3.6) denote that if a product \( p \) is allocated to line \( j \) at period \( n \), at most one product \( p' \) is processed right after and/or before it.

\[
\sum_{p' \in P_j, p' \neq p} X_{p', p, j, n} \leq Y_{p, j, n} \quad \forall p \in P_j, j, n \tag{3.5}
\]

\[
\sum_{p' \in P_j} X_{p, p', j, n} \leq Y_{p, j, n} \quad \forall p \in P_j, j, n \tag{3.6}
\]

According to constraint set (3.7) the total number of active sequencing variables \( X_{p, p', j, n} \) plus the line utilization binary variable \( V_{j, n} \) should equal the total active binary variables \( Y_{p, j, n} \) in a packing line at period \( n \). For example, if five products are processed in packing line \( j \) then four sequencing variables will be active.

\[
\sum_{p \in P_j} \sum_{p' \in P_j, p' \neq p} X_{p', p, j, n} + V_{j, n} = \sum_{p \in P_j} Y_{p, j, n} \quad \forall j, n \tag{3.7}
\]

Constraint set (3.8) forces a product \( p' \) that follows another product \( p \) on a packing line \( j \) (i.e. \( X_{p, p', j, n} = 1 \)) to start after the completion time of product \( p \) plus the required changeover time between them. A lower bound is imposed on completion time \( C_{p, j, n} \) of each product by constraint set (3.9). In particular, the completion time has to be greater than the setup time of the packing line, the required processing time and the changeover time to any predecessor \( p' \). Constraints 3.10 ensure that the horizon of each production time period is not violated. The completion time of every packing process must be less than the available period production time.

\[
C_{p, j, n} + sdf_{j, p, p'} \leq C_{p', j, n} - T_{p, j, n} + M \cdot (1 - X_{p, p', j, n}) \quad \forall p \in P_j, p' \in P'_j, p' \neq p, j, n \tag{3.8}
\]

\[
C_{p, j, n} - T_{p, j, n} \geq su_{j, n} \cdot Y_{p, j, n} + \sum_{p' \in P_j, p' \neq p} sdf_{j, p, p'} \cdot X_{p', p, j, n} \quad \forall p \in P_j, p' \in P'_j, p' \neq p, j, n \tag{3.9}
\]

\[
C_{p, j, n} \leq hor_{j, n} \cdot Y_{p, j, n} \quad \forall p \in P_j, j, n \tag{3.10}
\]

Inventory and capacity constraints

Constraints (3.11) impose the lower and upper bounds of the inventory level of product \( p \) at every production period \( n \).

\[
s_{p, n}^{\min} \leq S_{p, n} \leq s_{p, n}^{\max} \quad \forall p, n \tag{3.11}
\]
The inventory of product \( p \) at period \( n \), \( S_{p,n} \), is the summation of the previous period inventory \( S_{p,n-1} \), plus the total production \( Q_{p,j,n} \), minus the product demand \( dem_{p,n} \).

\[
S_{p,n} = S_{p,n-1} + \sum_{j \in J_p} Q_{p,j,n} - dem_{p,n} \quad \forall \ p,n
\] (3.12)

To obtain feasible production schedules, the batch stage (sterilization) must be included in the mathematical model. Constraints (3.13) state that the total produced number of products \( p \) must be within allowed limits based on the production capacity \( st_{p}^{cap} \) of each sterilizer. Binary variable \( YS_{st,n} \) denotes that a sterilization chamber \( st \) is active in period \( n \).

\[
\sum_{j \in J_p} Q_{p,j,n} \leq \sum_{st} YS_{st,n} \cdot st_{p}^{cap} \quad \forall \ p,n
\] (3.13)

**Tightening constraints**

The summation of the processing times of all products being processed in packing line \( j \), plus the required changeovers and the setup time of the unit, must be less than the scheduling horizon (24 hours) of each period \( n \).

\[
\sum_{p \in P} T_{p,j,n} + \sum_{p \in P_j} \sum_{p' \in P, p' \neq p} sdf_{j,p,p'} \cdot X_{p,p',j,n} \leq \text{hor}_{j,n} - su_{j,n} \cdot V_{j,n} \quad \forall \ j,n
\] (3.14)

**Objective**

The objective function of the model is expressed by constraints (3.15), which is the minimization of the total production makespan \( C_{max} \), hence the minimization of changeovers and unnecessary idle times.

\[
C_{max} \geq C_{p,j,n} \quad \forall \ p \in P_j, j,n
\] (3.15)

**Nomenclature**

**Indices/sets**

- \( p, p' \in P \) \( p \), \( p' \) are products
- \( j \in J \) \( j \) is a packing line
- \( st \in ST \) \( st \) is a sterilizer
- \( n \in N \) \( n \) is a scheduling period – days

**Subsets**

- \( J_p \) units that are able to pack products \( p \)
- \( P_j \) products \( p \) that can be processed in packing line \( j \)
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Parameters

\( b_{p,j,n}^{\text{max}} \)  
maximum production run (in cans) possible

\( \text{rate}_{p,j} \)  
packing rate of product \( p \) in packing line \( j \) (in cans/hour)

\( \text{hor}_{j,n} \)  
scheduling horizon of unit \( j \) at day \( n \) (in hours)

\( \text{su}_{j,n} \)  
ssetup time of unit \( j \) at day \( n \) (in hours)

\( \text{sd}_{j,p,p'} \)  
changeover time between products \( p \) and \( p' \) in unit \( j \) (in hours)

\( M \)  
big-M parameter (available scheduling horizon)

\( \text{dem}_{p,n} \)  
demand for product \( p \) at day \( n \) (in cans)

\( s_{p}^{\text{min}} \)  
minimum allowed inventory of product \( p \) (in cans)

\( s_{p}^{\text{max}} \)  
maximum allowed inventory of product \( p \) (in cans)

\( \text{cap}_{st} \)  
maximum available production capacity of sterilizers (in cans)

Binary Variables

\( Y_{p,j,n} \)  
allocation variable of product \( p \) to packing line \( j \) in time period \( n \)

\( V_{j,n} \)  
variable signifying that a packing line \( j \) is active in period \( n \)

\( X_{p,p',j,n} \)  
immediate precedence variable: \( = 1 \) if product \( p \) is processed before product \( p' \) in packing line \( j \) at period \( n \)

\( Y_{st,n} \)  
signifying that a sterilization unit \( st \) is active at day \( n \)

Continuous Variables

\( C_{p,j,n} \)  
completion time for product \( p \) in unit \( j \) in day \( n \) (in hours)

\( T_{p,j,n} \)  
required packing time for product \( p \) in unit \( j \) in day \( n \) (in hours)

Integer Variables

\( Q_{p,j,n} \)  
amount of product \( p \) produced in unit \( j \) in time period \( n \) (in cans)

\( S_{p,n} \)  
inventory level of product \( p \) in time period \( n \) (in cans)

Free Variable – Objective

\( C_{\text{max}} \)  
makespan (in hours)
3.3 Validation results

An industrial case for the weekly scheduling of 150 final products is demonstrated. Realistic data have been provided by FRINSA. The MILP model presented in section 3.2 was implemented in GAMS 24.9 [20] and solved using CPLEX 12.0. The problem instance was solved to optimality in reasonable computational time. In total the production of more than 15 million cans was successfully produced, representing a really demanding production week for the FRINSA plant. The maximum production makespan was 22.6 hours in line P2_L1 on Monday. Figure 3.2 illustrates the active times (processing plus setup and changeover times) for all packing lines at every period (day). It is evident from the utilization of the packing lines that the packing stage is not the production bottleneck of the plant. Despite, successfully satisfying the demand, most of the packing lines are underutilized. Therefore, an important conclusion is that it is essential to include the previous production stages (mainly the sterilization) to get a feasible production schedule for the FRINSA plant.

![Figure 3.2: Utilization of packing lines](image)

4 Modelling approach B: Multistage semi-continuous production

The results of solving the scheduling problem of FRINSA using approach A underlined the necessity of integrating the sterilization stage in the optimization procedure. Based on these observations approach B is constructed. Section 4 is structured as follows. First, the basic concepts of approach B are specified (section 4.1) and a MIP-based solution strategy to solve the multi-stage scheduling problem is presented (section 4.2). Finally, the solution strategy is validated, considering a real problem instance of the FRINSA plant (section 4.3).
4.1 Problem statement

In this approach both the packing and sterilization stages are modelled in detail, thus a multi-stage semi-continuous (mixed batch and continuous) production is studied. In particular, the sterilization (batch) and packing (continuous) stage of the plant are studied. Hence, this approach considers the production scheduling problem of industrial-scale multiproduct multistage semi-continuous processes.

The problem addressed here is formulated based on the following items:

- A set of processing stages \( s \in S \) (\( s_1 \) = sterilization stage, \( s_2 \) = packing stage).
- A set of processing units \( j \in J \). \( J_p \) are the units that can process product \( p \), while \( J_s \) the units that can process stage \( s \).
- A set of products \( p \in P \) with specific production targets \( \text{dem}_p \), packing rates \( \text{rate}_p \), sterilization times \( t^\text{ster}_p \), minimum inventory limitations \( s^\text{min}_p \) and initial inventories \( s^\text{init}_p \). \( P^a \) are the products that can be processed in each unit \( j \). \( P^a \) is the subset of products to be optimized.
- A set of batches \( b \in B \). The total demand of product \( p \) is divided into batches, based on the capacity of the sterilization chambers \( BC_p \).
- A changeover operation of time \( \text{ch}_{p,p',j} \) is necessary in each packing line whenever the production is changed between two products. No changeover exists between products in the sterilization stage.

Technically in the packing stage (continuous) product-lots are processed. In the mathematical representation though there is no need to distinguish between a product-batch in the batch stage, and a product-lot (which is processed in continuity to the previously mentioned product-batch) in the continuous stage. Therefore, for the sake of simplicity, the term batch is used in both the sterilization and the packing stage.

We assume a non-preemptive operation, no fluctuations in orders (demand uncertainty) and that there are no limitations of resources (manpower, steam, water, etc.). Additionally, it is assumed than a zero-wait policy between the two processing stages is followed. In reality, there is the possibility to introduce some wait between the stages of the FRINSA plant. Taken into account this buffer, will lead to a better schedule, but will also radically increase the complexity of the problem. Thus, a zero-wait policy is chosen in this stage of the study.

The key decision variables are:

- the required number of product-batches \( p \cdot b \) to be scheduled to satisfy the demand, \( NB_p \)
- the allocation of product-batches \( p \cdot b \) to units in every stage \( s \), \( Y_{p,b,s,j} \)
- the relative sequence of any pair of product-batches \( p \cdot b \) and \( p' \cdot b' \) in the sterilization stage \( X_{p,b,p',b'} \)
- the relative sequence of any pair of products \( p \) and \( p' \) in the packing stage \( X_{p,p'} \)
- the completion \( C_{p,b,s} \) and starting times \( L_{p,b,s} \) of each product-batch in all stages
4.2 Modelling Approach

In this approach a scheduling problem of significant degree of complexity, in terms of number of products, shared resources, stages and multiple operational and quality constraints is examined. Hence, a holistic approach similar to the one studied in the single stage process in approach A, will not suffice. To get a feasible solution in reasonable time, the initial problem needs to be split into a set of tractable subproblems. Therefore, an MILP-based solution strategy consisting of the following components is adopted:

- A pre-optimization step for the batching decisions.
- A MILP-model for the scheduling (unit allocation, timing and sequencing) decisions.
- A product-based decomposition strategy to reduce the initial intractable problem into tractable subproblems.

In figure 4.1 the basic rationale behind the decision making of the proposed strategy is illustrated.

4.2.1 Preoptimization step

Goal of this step is to convert the product orders into batches in the sterilization stage to fully satisfy the given demand (figure 4.2). In most food industries, such as the one studied in this project, the industrial practice imposes operations of the intermediate batch processes in their maximum capacity. The maximum utilization of the batch stage, allows for high production levels and minimization of changeovers between products. Thus, the batching problem can be solved a priori.
Equation (4.1) defines the minimum number of batches, $NB_p$, to fully satisfy the demand. This number depends on the demand, the capacity of the sterilization chamber, the initial inventory at the start of the scheduling horizon and the minimum allowed inventory. Note that all sterilization units in the FRINSA plant are identical and the capacity depends only on the type of product (size of can).

$$NB_p = \left\lceil \frac{dem_p - s^\text{init}_p + s^\text{min}_p}{BC_p} \right\rceil \quad \forall \ p \in P^{\text{in}}$$  \hspace{1cm} (4.1)

To avoid overproduction the parameter $LBC_p$ is introduced. This item specifies the amount of product being processed in the last product-batch. The value of this parameter depends on the amount that needs to be produced (demand plus minimum inventory minus initial inventory) and the amount that has already been processed in all the previous batches of the product.

$$LBC_p = dem_p - s^\text{init}_p + s^\text{min}_p - BC_p \cdot (NB_p - 1) \quad \forall \ p \in P^{\text{in}}$$  \hspace{1cm} (4.2)

Based on the previous calculations and the given packing rate, the required time to process every product-batch in the packing stage is calculated.

$$t^\text{pack}_{p,b} = \frac{BC_p}{rate_p} \quad \forall \ p \in P^{\text{in}}, b \neq b_{\text{last}}$$  \hspace{1cm} (4.3)

$$t^\text{pack}_{p,b} = \frac{LBC_p}{rate_p} \quad \forall \ p \in P^{\text{in}}, b = b_{\text{last}}$$  \hspace{1cm} (4.4)

In order to clearly understand this step of the solution strategy, assume a product with demand of 1200 cans, capacity 500 cans per sterilizer and for simplicity, zero minimum and initial inventory. To satisfy the demand, a total number of 3 batches is required. The first 2 batches will fully utilize the sterilization chambers (500cans), while the last batch will process just 200cans.
4.2.2 MILP-model

In this section the MILP formulation is presented for the multiproduct multistage semi-continuous facility of FRINSA. The MILP model employed in this approach is based on an extension of the general precedence framework as originally introduced by our previous work [18]. The constraints are grouped according to the type of decision.

Unit allocation

Constraint set (4.5) ensures that each product-batch \( p \)-\( b \) will go through one unit \( j \) in every stage \( s \), utilizing the binary variable \( Y_{p,b,s,j} \):

\[
\sum_{j \in (J_p \cap J_s)} Y_{p,b,s,j} = 1 \quad \forall p \in P^n, b \in B, s \in S
\]  
(4.5)

Timing constraints

The timing constraints (4.6) and (4.7) are imposed for a product-batch in the same production stage. In particular, constraints (4.6) ensure that the completion time of a product-batch in the sterilization stage is equal to the starting time of the process plus the required sterilization time for the specific product. Same holds for constraints (4.7), which express the connection between the start and the finish of the packing process based on the required packing time for each product-batch.

\[
L_{p,b,s} + t_{st} = C_{p,b,s} \quad \forall p \in P^n, b \in B, s \in S : b \leq NB_p, s = 1
\]  
(4.6)

\[
L_{p,b,s} + t_{pk} = C_{p,b,s} \quad \forall p \in P^n, b \in B, s \in S : b \leq NB_p, s = 2
\]  
(4.7)

Constraints (4.8) guarantee that a zero-wait policy is adopted. Specifically, it imposes that the packing process of a product-batch should start exactly when the sterilization process is completed. In constraint set (4.9) the same policy is ensured, by interconnecting the starting time of a product-batch in the two processing stages. The FRINSA plant follows a single production campaign policy in the packing stage, without allowing a waiting time between batches of the same product. This operational constraint is imposed by (4.10). According to these constraints, the completion time for a product batch should be equal to the staring time of the next batch of the same product.

\[
L_{p,b,s} = C_{p,b,s-1} \quad \forall p \in P^n, b \in B, s \in S : b \leq NB_p, s = 2
\]  
(4.8)

\[
L_{p,b,s-1} + t_{st} = L_{p,b,s} \quad \forall p \in P^n, b \in B, s \in S : b \leq NB_p, s = 2
\]  
(4.9)

Sequencing constraints

The sequencing constraints between product-batches in both the sterilization and packing stage are portrayed in constraints (4.11) – (4.14). In particular, constraints (4.11) and (4.12) enforce the starting time of a product-batch \( L_{p'p',s} \) to be greater than the completion time of a product-batch \( C_{p,b,s} \) processed beforehand, if and only if both batches are allocated to the same sterilization chamber. Similarly, constraints (4.13) define the sequencing of different products in the packing line. Constraints (4.12) and (4.14) are the supplementary versions of (4.11) and (4.13) accordingly. Two
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general precedence binary variables are employed \( X_{p,p'} \) and \( \overline{X}_{p,b,p',b'} \). Due to the single campaign policy, imposed by constraint (4.10), it is not necessary to declare a sequencing variable between product-batches in the packing stage. Hence, it is sufficient to just define the sequencing between products in the packing lines. This approach drastically reduces the number of binary variables to be specified in the MILP model.

\[
L_{p,b,s} > C_{p,b,s} - M \cdot \left(1 - X_{p,b,p',b'} \right) - M \cdot \left(2 - Y_{p,b,s,j} - Y_{p',b',j,s} \right)
\]
\( \forall p \in P^m, p' \in P^m, b, b' \in B, s \in S, j \in (J_p \cap J_{p'} \cap J_s): p < p', b \leq NB_p, b' \leq NB_{p'}, s = 1 \)  \( (4.11) \)

\[
L_{p,b,s} > C_{p,b,s} - M \cdot \overline{X}_{p,b,p',b'} - M \cdot \left(2 - Y_{p,b,s,j} - Y_{p',b',j,s} \right)
\]
\( \forall p \in P^m, p' \in P^m, b, b' \in B, s \in S, j \in (J_p \cap J_{p'} \cap J_s): p < p', b \leq NB_p, b' \leq NB_{p'}, s = 1 \)  \( (4.12) \)

\[
L_{p,b,s} > C_{p,b,s} + ch_{p,b,s,j} - M \cdot \left(1 - X_{p,b,p'} \right) - M \cdot \left(2 - Y_{p,b,s,j} - Y_{p',b',j,s} \right)
\]
\( \forall p \in P^m, p' \in P^m, b, b' \in B, s \in S, j \in (J_p \cap J_{p'} \cap J_s): p < p', b \leq NB_p, b' \leq NB_{p'}, s = 2 \)  \( (4.13) \)

\[
L_{p,b,s} > C_{p,b,s} + ch_{p,b,s,j} - M \cdot \overline{X}_{p,b,p'} - M \cdot \left(2 - Y_{p,b,s,j} - Y_{p',b',j,s} \right)
\]
\( \forall p \in P^m, p' \in P^m, b, b' \in B, s \in S, j \in (J_p \cap J_{p'} \cap J_s): p < p', b \leq NB_p, b' \leq NB_{p'}, s = 2 \)  \( (4.14) \)

To avoid symmetric solutions, if two batches of the same product are assigned to the same unit, we assume that the lower indexed batch is performed first, according to (4.15)

\[
L_{p,b,s} > C_{p,b,s} - M \cdot \left(2 - Y_{p,b,s,j} - Y_{p',b',j,s} \right)
\]
\( \forall p \in P^m, b, b' \in B, s \in S: b \leq NB_p, b' \leq NB_{p'}, s = 2 \)  \( (4.15) \)

**Objective**

Similar to the previous approach, the objective is the minimization of the total production makespan, hence the minimization of total changeover time and unnecessary idle times.

\[
C_{\max} \geq C_{p,b,s} \quad \forall p \in P^m, b \in B, s \in S: b \leq NB_p, s = 2
\]  \( (4.16) \)

**Nomenclature**

**Indices/Sets**

- \( p, p' \in P \) products
- \( b, b' \in B \) product batches
- \( j \in J \) processing units
- \( s \in S \) processing stages

**Subsets**

- \( P_j \) products \( p \) that can be processed in unit \( j \)
- \( J_p \) available units \( j \) to process product \( p \)
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\( J_s \) available units \( j \) to process stage \( s \)

\( P_{jn} \) subset of products \( p \) included in the optimization

**Parameters**

\( t_{ster}^p \) required sterilization time for product \( p \) (in hours)

\( t_{pack}^{p,b} \) packing time for product-batch \( p, b \) (in hours)

\( LBC_p \) capacity of the last batch of product \( p \) (in cans)

\( BC_p \) capacity of batches of product \( p \) (in cans)

\( NB_p \) number of batches required to satisfy demand of product \( p \)

\( M \) big-M parameter (available scheduling horizon)

\( dem_p \) demand for product \( p \) (in cans)

\( s_{min}^p \) minimum allowed inventory of product \( p \) (in cans)

\( s_{init}^p \) initial inventory of product \( p \) (in cans)

\( rate_p \) packing rate of product \( p \) (in cans)

\( ch_{p,p',j} \) changeover time between product \( p \) and \( p' \) in unit \( j \) (in hours)

**Binary Variables**

\( Y_{p,b,s,j} \) allocation variable of product-batch \( p, b \) in unit \( j \) of processing stage \( s \)

\( X_{p,p'} \) general precedence variable: \( =1 \) when product \( p \) is processed before product \( p' \) in the packing stage

\( \overline{X}_{p,b,p',b'} \) general precedence variable: \( =1 \) when product-batch \( p, b \) is processed before product-batch \( p', b' \) in the sterilization stage

**Continuous Variables**

\( C_{p,b,s} \) completion time of product-batch \( p, b \) in processing stage \( s \) (in hours)

\( L_{p,b,s} \) starting time of product-batch \( p, b \) in processing stage \( s \) (in hours)

**Free Variable – Objective**

\( C_{max} \) makespan (in hours)
4.2.3 Decomposition algorithm

The goal of the decomposition strategy is to decompose the large-scale scheduling problem into a subset of the involved products. This way the search space for the solver is reduced and computational time is favored. In particular, in each iteration of the algorithm a predefined number of products \( P^{in} \) are scheduled, until a complete schedule for all products is extracted. Basic parameters of the decomposition algorithm are the order of the last scheduled product \( z \) and the number of products to be scheduled at each iteration step \( S_d \). After the resolution of the MILP model at each iteration, allocation, sequencing and timing variables for the already scheduled product orders are fixed. A pseudocode describing the decomposition algorithm is found below:

1. **Initialize** \( z \) and step parameters
2. **For** \( z = 1 \) to number of products to be scheduled with step \( S_d \)
   - **Loop over all products**
     - **If** \( \text{order}(p) \leq z \)
       - \( P^{in} = \text{YES} \)
     - **End if**
   - **End loop**
3. **Solve MILP model**
4. **Fix** \( Y_{p,b,s,j} \), \( X_{p,p',b'} \), \( X_{p,b,s,j} \), \( C_{p,b,s} \), \( L_{p,b,s} \) for all \( p \in P^{in} \)

Figure 4.3 illustrates the flowchart of the complete solution strategy algorithm. Initially, the batching decisions are made in the preoptimization step described in section 4.2.1. Afterwards, the decomposition algorithm parameters are defined and the subproblem is solved utilizing the aforementioned MILP model (section 4.2.2). Next, the derived decisions made for the specific subproblem are fixed and the algorithm continues to the next iteration. Finally, when all products are considered, the final schedule is generated.

![Solution Strategy Flowchart](image)

4.3 Validation results

The applicability and efficiency of the proposed modelling framework and solution strategy is illustrated in a real problem instance of the FRINSA plant. Both the sterilization and packing processes of the
plant are modelled in detail, while 100 final products are to be scheduled. The preoptimization step calculates that 362 batches are required in total to satisfy the demand. Utilizing the proposed decomposition technique, 20 final products are scheduled in each iteration. This number is chosen as a trade-off between the total computational time and the solution value. Lower values decrease immensely the computational complexity of the problem in hand, but also produce worse results for the complete final schedule. Respectively, choosing to schedule a larger number of products at each iteration proves to be computationally unacceptable. The proposed solution strategy was implemented in GAMS 24.9 [20] and solved using CPLEX 12.0. Optimality is reached for all iterations of the suggested strategy. Figure 4.4 illustrates the complete schedule generated for the sterilization units, while in figure 4.5 the corresponding schedule for the packing units is depicted.

It is evident from the Gantt charts that the sterilizers are more active than the packing units. On the other hand, utilization of the packing lines depends on the demand profile, since each product can be processed only at specific packing units, whereas all products can be processed in any sterilizer. Nevertheless, all packing units are underutilized compared to the sterilizers. Hence, the sterilization stage is identified as the production’s bottleneck in line with conclusions presented in the previous chapter. This is also illustrated in figure 4.6, where the utilization of all units is shown.

Despite the ability of the suggested method to provide a complete schedule for a complex problem instance of the FRINSA plant, some limitations are also revealed. The total computational time for the presented case was 4 hours. Hence, the model is incompetent of scheduling more demanding weeks or it should be extended to include all the processing stages in reasonable time.

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Figure 4.4: a) Gantt chart of the sterilization units, b) indicative schedule with product codes for the ST7-11 sterilization units

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Following the results of the previous works, a third solution approach to solve the complex scheduling problem of the FRINSA plant is developed. Similar to the approach presented in section 4, the processing stages of sterilization and packing are modelled in detail. A main difference is the categorization of products into product families, which share similar characteristics in the sterilization stage. This action immensely reduces the number of choices (decision variables) to be made in the sterilization process of the plant. Thus, a significant decrease in the computational complexity of the problem in hand is achieved.

5 Modelling approach C: Multistage semi-continuous production utilizing product families

In this chapter, the multi-stage multi-product semi-continuous canned-fish plant of FRINSA is addressed. In particular, the processing stages of sterilization and packing are considered.

The problem addressed is formulated according to the following items:
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- A set of processing stages \( s \in S \) (\( s_0 \) = sterilization stage, \( s_1 \) = packing stage).
- A set of processing units \( j \in J \) with setup time \( s^j \). \( J \) are the units that can process product \( p \), while \( J_s \) are the units that can process stage \( s \).
- A set of product families \( f, f' \in F \) with capacity in each sterilization chamber \( BC_f \) and sterilization time \( t^s_f \). \( F^m \) is the subset of family products to be optimized.
- A set of products \( p \in P \) with specific production targets \( dem_p \), and packing rates in every unit \( j \). \( P \) are the products that can be processed in each unit \( j \). \( P^m \) is the subset of products to be optimized and \( P_j \) are the products that belong to family \( f \).
- A set of family batches \( b \in B \). \( B_p \) is the subset of family-batches that process a specific product \( p \).
- A changeover operation of time \( ch_{p,p',j} \) is necessary in each packing line whenever the production is changed between two products. No changeover exists between product families in the sterilization stage.

We assume a non-preemptive operation, no fluctuations in orders (demand uncertainty) and that there are no limitations of resources (manpower, steam, water, etc.). Additionally, a zero-wait policy between the two processing stages is assumed.

Notice that product families are defined only for the sterilization stage, while products only for the packing stage. The connection between the two stages is done through the individual batches, which contain products of a specific family. The concept of a batch is slightly different compared to the previous approach B. In the sterilization stage the term family-batch is used, while the term product-batch is used in the packing stage. This concept will be further explained in the next section.

The key decision variables are:
- the required number of family-batches \( f - b \) to be scheduled to satisfy the demand, \( NBF_f \)
- the allocation of products \( p \) in every family-batch \( f - b \), \( Y_{p,b} \)
- the total amount of product \( p \) processed in each batch \( b \), \( Q_{p,b} \)
- the allocation of family-batches \( f - b \) in the sterilization stage, \( YF_{f,b,j} \)
- the allocation of product-batches \( p - b \) in the packing stage, \( YP_{p,b,j} \)
- the relative sequence between families \( f \) and \( f' \) in the sterilization stage, \( XF_{f,f'} \)
- the relative sequence between products \( p \) and \( p' \) in the packing stage, \( XP_{p,p'} \)
- the completion times of family-batches \( CF_{f,b} \) in the sterilization stage and product-batches \( CP_{p,b} \) in the packing stage

Our goal is to reduce the total production time \( C_{\text{max}} \) as much as possible while fully satisfying the customers’ demand.

Note, that no inventory related parameters and variables are defined. After thorough discussions with the plant operators it was concluded that there are no significant inventory limitations for each product in the FRINSA plant. Therefore, all related items are discarded. The only constraint that
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needs to be considered is the total storage capacity of the plant, which will not significantly affect the search space for the solver and will be included in later versions of the model.

5.2 Modelling approach

Production scheduling in food industries typically deal with a very large number of products. To our favor, many products appear similar characteristics. This is also the case in the FRINSA plant, where products share same attributes in the sterilization stage. Therefore, these products can be treated as a product family group (family). This idea has been successfully implemented in [16], but for a single stage process. Because of this modelling concept, the size of the underlying model, thus the computational effort required to solve it, is reduced. Notice that this is achieved without sacrificing any feasibility constraint. In the presented approach, products belong in the same family if and only if:

- They share the same batch capacity $BC_f$.
- They require equal sterilization time $t^{ster}_f$.
- They can be sterilized simultaneously in the same sterilization chamber.

Due to the increased complexity of the scheduling problem, an MIP-based solution strategy is adopted, consisting of the following:

- An MILP model for the batching decisions (batch optimizer).
- A continuous general precedence based MILP model for the unit allocation, timing and sequencing decisions (Scheduling-MILP).
- A decomposition strategy.

The figure below illustrates an overview of the decision-making approach in the suggested solution strategy.

![Figure 5.1: Decision making](image)

As mentioned in the problem statement, an important concept of the modeling approach is the definition of family-batches in the sterilization stage and product-batches in the packing stage. While, the products are categorized in product families, this holds true only for the sterilization process. After the completion of the sterilization process, the various products being processed in a sterilization chamber, need to be packaged separately in the packing stage. Therefore, all the
decisions in the packing stage needs to be done in terms of products and not families. The set of batches allow us to connect the two stages and provide a continuity between them. An illustrative example can be seen in figure 5.2. Let us assume a family F01, in which products P01, P02 and P03 belong. As it can be seen in the figure, the total demand of the family requires four batches to be processed in the sterilization stage. Thus, four family-batches F01.B01, F01.B02, F01.B03 and F01.B04 need to be processed in the sterilization stage. But each family-batch does not consist of just one product. For example, family-batch F02.B02 consists of products P01 and P02. These have to be separately processed in the continuous stage of packing. So, in the packing stage a total of 6 product-batches must be processed, mainly P01.B01, P01.B02, P02.B02, P02.B03, P03.B03 and P03.B04. So, defining just one set of batches b, both family-batches in the sterilization stage and product-batches in the packing stage can be expressed.

![Figure 5.2: Definition of family-batches and product-batches](image)

### 5.2.1 Batch optimizer

Goal of the batch optimizer is to successfully distribute the product orders \( p \) into family-batches \( f - b \), so that the given demand is fully satisfied. Initially, some preoptimization calculations need to be made. The total demand of a product family \( \text{dem}_f \) equals the summation of the demands given for all products \( \text{dem}_p \) belonging in this family (5.1). The total number of family-batches \( \text{NB}_f \) to satisfy the demand, can be calculated through the total demand of the family and the quantity that can be processed in each batch (5.2).

\[
\text{dem}_f = \sum_{p \in (f \cap P)} \text{dem}_p, \quad \text{where } f \in F^{in} \tag{5.1}
\]

\[
\text{NB}_f = \text{NBF}_f = \frac{\text{dem}_f}{BC_f}, \quad \text{where } p \in (P_f \cap P^{in}), f \in F^{in} \tag{5.2}
\]

Constraint set (5.3) ensures that the total amount of product processed in all family-batches will fully satisfy the given demand.

\[
\sum_{b \in \text{NB}_f} Q_{p,b} \geq \text{dem}_p \quad \forall p \in P^{in} \tag{5.3}
\]

The industrial policy adopted in most food industries is that the intermediate batch stages are fully utilized, which leads to higher productivity. According to this policy, constraints (5.4) are implemented in the model. More specifically, the total amount of product produced in every family-batch needs to be equal to the capacity of the sterilization chambers \( BC_f \).
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\[ BC_f = \sum_{p \in F_f} Q_{p,b} \quad \forall f \in F^m, b \leq NBF_f \] (5.4)

Constraints (5.5) ensure that the produced amount of a product \( p \) in every family-batch \( Q_{p,b} \) is not exceeding the capacity constraints imposed by the sterilization chambers. In particular, if a product is processed in a family batch \( \overline{Y}_{p,b} \), then the processed amount \( Q_{p,b} \) needs to be at least one can and at most be equal to the maximum capacity of the sterilization unit.

\[ \overline{Y}_{p,b} \leq Q_{p,b} \leq BC_f \cdot \overline{Y}_{p,b} \quad \forall p \in (P_f \cap \mathcal{P}^m), f \in F^m, b \in B : b \leq NB_p \] (5.5)

Goal of this optimization step is to process every product in the least possible number of batches (constraint 5.6). This number has a significant impact to the next optimization step (Scheduling MILP), since distributing the products in more family-batches, leads to more product-batches to be scheduled in the packing stage, thus larger MILP-models.

\[ \text{mix} = \sum_p \sum_b \overline{Y}_{p,b} \] (5.6)

### 5.2.2 Scheduling MILP

In the next step, all scheduling (unit allocation, timing and sequencing) decisions for both the sterilization and the packing stage are optimized. The MILP model employed in this approach is based on the general precedence framework [6]. The constraints are grouped according to the type of decision.

#### Unit Allocation

Constraint set (5.7), guarantees that each family-batch \( f - b \) is processed by exactly one sterilization chamber. Similarly, constraints (5.8) ensure that all product-batches \( p - b \) go through just one packing line.

\[ \sum_{j \in \mathcal{J}} Y_{F_{f,b,j}} = 1 \quad \forall f \in F^m, b \in B : b \leq NBF_f, s = 1 \] (5.7)

\[ \sum_{j \in (J_{p,b,j})} Y_{P_{p,b,j}} = 1 \quad \forall p \in \mathcal{P}^m : s = 2 \] (5.8)

#### Timing constraints

Constraints (5.9) and (5.10) impose the timing constraints for all batches. In particular, constraint set (5.9) ensures that the completion time of a family-batch \( f - b \) in the sterilization stage \( CF_{f,b} \), will be larger than the required sterilization time of the process \( t^\text{ster}_f \). Respectively, constraints (5.10) guarantee that the finish of the packing process of a batch \( CP_{p,b} \) will be equal to the completion time for the sterilization process \( CF_{f,b} \) plus the necessary packing time \( t^\text{pack}_{p,b,j} \) and any waiting time \( W_{p,b} \) between the stages. In the context of this work, no waiting time is allowed.

\[ CF_{f,b} \geq \sum_{j \in \mathcal{J}} t^\text{ster}_f \cdot Y_{F_{f,b,j}} \quad \forall f \in F^m, b \in B : b \leq NBF_f \] (5.9)
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\[ CP_{p,b} - \sum_{j\in(J_{p}\cap J_{s})} (t^{pack}_{p,b,j} \cdot YP_{p,b,j}) = CF_{f,b} + W_{p,b} \quad \forall p \in (P^{in} \cap P_{f}), f \in F^{in}, b \in B_{p} \quad (5.10) \]

Sequencing constraints

Constraint set (5.11) states that the completion time of a family-batch \( f' - b' \) that follows another family-batch \( f - b \) in a sterilization chamber is greater than the completion time of \( f - b \) plus the required sterilization time \( t_{f,b}^{ster} \). Accordingly, (5.13) impose that the completion time of a product-batch \( p' - b' \) in the packing stage following another product-batch \( p - b \) in the same packing line, has to be greater than the completion time of \( p - b \) plus the necessary packing time \( t_{p,b}^{pack} \) and the required changeover time between them \( ch_{p,p',j} \). Constraint sets (5.12) and (5.14) are complementary to (5.11) and (5.13) accordingly.

\[ CF_{f,b} \leq CF_{f',b'} - t_{f}^{ster} \cdot YF_{f',b',j} + M \cdot (1 - X F_{f',j}) + M \cdot (2 - Y F_{f,b,j} - Y F_{f',b',j}) \]
\[ \forall f \in F^{in}, f' \in F^{in}, b, b' \in B, j \in J_{s} : f < f', b \leq NBF_{f}, b' \leq NBF_{f'}, s = 1 \]  
\[ (5.11) \]

\[ CF_{f',b'} \leq CF_{f,b} - t_{f}^{ster} \cdot YF_{f,b,j} + M \cdot X F_{f',j} + M \cdot (2 - Y F_{f,b,j} - Y F_{f',b',j}) \]
\[ \forall f \in F^{in}, f' \in F^{in}, b, b' \in B, j \in J_{s} : f < f', b \leq NBF_{f}, b' \leq NBF_{f'}, s = 1 \]  
\[ (5.12) \]

\[ CP_{p,b} + ch_{p,p',j} \leq CP_{p',b'} - t_{p,b}^{pack} \cdot YP_{p',b',j} + M \cdot (1 - X P_{p',j}) + M \cdot (2 - Y P_{p,b,j} - Y P_{p',b',j}) \]
\[ \forall p \in P^{in}, p' \in P^{in}, b \in B_{p}, b' \in B_{p'}, j \in (J_{p} \cap J_{p'} \cap J_{s}) : p < p', s = 2 \]  
\[ (5.13) \]

\[ CP_{p',b'} + ch_{p',p,j} \leq CP_{p,b} - t_{p,b}^{pack} \cdot YP_{p',b',j} + M \cdot X P_{p',j} + M \cdot (2 - Y P_{p,b,j} - Y P_{p',b',j}) \]
\[ \forall p \in P^{in}, p' \in P^{in}, b \in B_{p}, b' \in B_{p'}, j \in (J_{p} \cap J_{p'} \cap J_{s}) : p < p', s = 2 \]  
\[ (5.14) \]

In order to avoid symmetric solutions, if two batches of the same family are assigned to the same sterilizer, we assume that the lower indexed batch is processed first (5.15). The same holds for the product-batches in the packing stage (5.16).

\[ CP_{p,b} \geq CP_{p,b} + \text{pack}_{p,b,j}^{time} \cdot YP_{p,b,j} - M \cdot (2 - Y P_{p,b,j} - Y P_{p',b',j}) \]
\[ \forall p \in P^{in}, b \in B_{p}, b' \in B_{p}, j \in (J_{p} \cap J_{s}) : b < b', s = 2 \]  
\[ (5.15) \]

\[ CF_{f,b} \geq CF_{f,b} + t_{f}^{ster} \cdot YF_{f,b,j} - M \cdot (2 - Y F_{f,b,j} - Y F_{f,b',j}) \]
\[ \forall f \in F^{in}, b \in B, b' \in B, j \in J_{s} : b < b', s = 1 \]  
\[ (5.16) \]
Objective

Our goal is to minimize the required time to fully satisfy the demand, hence minimize the total production makespan.

\[ C_{\text{max}} \geq CP_{p,b} \quad \forall \ p \in P^{\text{in}}, b \in B_p \]  

(5.17)

Nomenclature

Indices/Sets

\( p, p' \in P \) products
\( f, f' \in F \) product families
\( b, b' \in B \) family batches
\( j \in J \) processing units
\( s \in S \) processing stages

Subsets

\( P_f \) products \( p \) belonging to family \( f \)
\( J_p \) units \( j \) that can process product \( p \)
\( J_s \) units \( j \) that belong to processing stage \( s \)
\( B_p \) subset of batches \( b \), defining in which family-batches \( b \) the product \( p \) is processed
\( P^{\text{in}} \) subset of products included in the optimization
\( F^{\text{in}} \) subset of families included in the optimization

Parameters

\( t_{\text{ster}}^p \) required sterilization time for product \( p \) (in hours)
\( t_{\text{pack}}^{p,b,j} \) packing time for product-batch \( p \ b \) in unit \( j \) (in hours)
\( BC_f \) capacity of batches of family \( f \) (in cans)
\( NBF_f \) number of batches required to satisfy demand of family \( f \)
\( NB_p \) number of batches required to satisfy demand of product \( p \)
\( M \) big-M parameter (available scheduling horizon)
\( \text{dem}_p \) demand for product \( p \) (in cans)
\( \text{demF}_f \) demand for product \( p \) (in cans)
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\[ \text{rate}_{p,j} \] packing rate of product \( p \) in unit \( j \) (in cans)

\[ \text{ch}_{p,p',j} \] changeover time between product \( p \) and \( p' \) in unit \( j \) (in hours)

**Binary Variables**

\[ \bar{Y}_{p,b} \] allocation variable: =1 when product \( p \) is processed in batch \( b \)

\[ YF_{f,b,j} \] allocation variable: =1 when batch \( b \) of family \( f \) is processed in unit \( j \)

\[ YP_{p,b,j} \] allocation variable: =1 when batch \( b \) of product \( p \) is assigned to unit \( j \)

\[ XF_{f,f'} \] general precedence variable: =1 when family \( f \) is processed before family \( f' \)

\[ XP_{p,p'} \] general precedence variable: =1 when product \( p \) is processed before product \( p' \)

**Continuous Variables**

\[ CF_{f,b} \] completion time of family-batch \( f, b \) in the sterilization stage (in hours)

\[ CP_{p,b} \] completion time of product-batch \( p, b \) in the packing stage (in hours)

\[ W_{p,b} \] wait time of product-batch \( p, b \) between the sterilization and packing stage (in hours)

**Integer Variables**

\[ Q_{p,b} \] processed amount of product \( p \) in batch \( b \) (in cans)

**Free Variables – Objectives**

\[ \text{mix} \] total number of product batches

\[ C_{\text{max}} \] makespan (in hours)

### 5.2.3 Decomposition algorithm

The goal of the decomposition strategy is to split the initial scheduling problem into several tractable subproblems, which can be solved by the proposed Scheduling-MILP model in reasonable time. In each subproblem a predefined number of product families are scheduled. Basic parameters of the decomposition algorithm are the position of the last scheduled family \( z \) and the number of families scheduled at each iteration step \( S_d \). To improve the efficiency of the decomposition strategy the families are scheduled based on a specific insertion policy. In particular, an earliest due date (EDD) rule is adopted. In case two families have the same due date, then priority is given to the one with less unit flexibility, meaning that families with less alternative units should be scheduled first. Similar approaches for simpler problems have been successfully implemented in [12] and [19]. A pseudocode describing the decomposition algorithm follows:
Define insertion policy
Initialize z

For z=1 to amount of families being scheduled with step=S
  Loop over all families
    If position(f) ≤ z
      F_{in} = YES
    End if
  End loop
Solve MILP model
Fix variables for all f ∈ F_{in}
End for

Figure 5.4 illustrates the flowchart of the solution strategy algorithm. Initially, the batching subproblem is solved and all batching decisions are fixed. Next, the insertion policy is defined as already described. Afterwards, the decomposition algorithm parameters are defined and the subproblem is solved utilizing the aforementioned Scheduling-MILP model. Subsequently, the derived decisions made for the specific subproblem are fixed and the algorithm continues to the next iteration. Finally, when all families are considered, the final schedule is generated.

5.3 Validation results

The results for a real problem instance of the FRINSA production plant is demonstrated in this section. Data from a real weekly production provided by FRINSA was used. A total of 102 final products categorized into 75 families have to be scheduled over two production processes (sterilization and packing) consisting of multiple parallel processing units. To fully satisfy the demand, more than 1000 batch operations are required to be scheduled. Utilizing the proposed decomposition technique, 1 product family is scheduled in each iteration. Thus, a quick MIP model
resolution is ensured. Notice that each family consists of multiple products, which are split into numerous batches. Hence, even scheduling families one at a time, requires the optimization of multiple jobs in each iteration.

The proposed models and the decomposition algorithm was implemented using GAMS 24.9 [20]. To solve the MILP problems the IBM ILOG CPLEX 12.0 solver was used in a desktop PC with an Intel Core i7-6700 @3.4Ghz and 16GB of RAM. Optimality was reached for all model iterations of the solution strategy. A feasible solution was provided in very low computational time (less than 10 minutes). The total makespan of the computed schedule is 110 hours, meaning that the proposed methodology is able to successfully schedule a real week of the FRINSA plant. The Gantt charts of the sterilization and packing stage are displayed in figures (5.5) and (5.6).
6 Planning of maintenance activities

6.1 Problem description

The P&G case covered in this contribution covers the P&G manufacturing plant located in Amiens, France. In this plant, the Soluble Unit Dose (SUD) sachets (also known as Pods) are produced in many different variants, for different purposes and markets. The plant consists of different unit operations, from the receiving and storing of raw materials to the packing and palletizing of the finished products. For each type of unit operation, there are several of them working in parallel in different production lines.

In this contribution, the optimization of maintenance is tackled. The main goal in this case is the development of a predictive maintenance concept, which can be implemented into the plant in Amiens. Currently P&G is using a time-based approach in which all components in the production lines are changed periodically after a fixed number of days, neglecting their real status.

In the plant in Amiens, the maintenance for a production line is performed during specific maintenance timeslots. These slots are determined during the production planning for the plant. In general, a four-hour long maintenance slot is incorporated into the production schedule for each production line every two weeks. During these slots, maintenance tasks can be carried out. A SAP system provides all tasks which are due to be changed in the next days and an expert does the concrete scheduling of the tasks by hand. Each maintenance task can only be carried out by one or multiple workers with specific skill sets (e.g. electricians, mechanics...). Due to the fact that only a limited number of workers with specific skills is available during a maintenance slot, the tasks have to be planned in such a way that they can be carried out by the available personal. In addition, tasks have different priorities which have to be taken into account. In general, no task with a lower priority is allowed to be carried out if another task with a higher priority can be performed instead.

This project aims to reduce the costs associated with maintaining and repairing some of the components by introducing accurate predictions for their lifetime acquired through data-analysis. In addition, the equipment availability should be extended by using an improved scheduling procedure for the resulting maintenance tasks through better utilization of the available resources with the goal to free up time reserved for maintenance for production.

In this part of the report, an approach for the improved scheduling is presented.

6.2 Modelling approach

The mathematic model for the scheduling problem presented under section 6 is based on the Bin Packing Problem [21]. Furthermore, a method was developed which utilizes this formulation but improves the result by handling the priorities in an improved fashion.

6.2.1 Bin packing problem

The Bin Packing Problem (BPP) is a problem from the field of combinatorial optimization. It is one the classic NP-complete problems [22]. The goal of the optimization is to find the minimum number of bins $M = \{1, \ldots, m\}$ with a given capacity $c$ so that each of $n$ given items $N = \{1, \ldots, n\}$ with a positive weight $w_j$ is assigned to one bin without exceeding the capacity of any bin [1]. For example, for six items with the weights 2, 18, 23, 32, 42, 80 and a bin capacity of 100 the assignment of 2, 18
and 80 to one bin and the other items to another bin would lead to an optimal solution of two bins, since for the sum of all weights (197) at least two bins are required to pack all items into bins.

A mathematical formulation can be given as follows [21]:

\[
\begin{align*}
\min \sum_{i=1}^{m} y_i \\
\text{s.t.} \sum_{j=1}^{n} w_j \cdot x_{ij} & \leq cy_i, \forall i \in M \\
\sum_{i=1}^{m} x_{ij} & = 1, \forall j \in N
\end{align*}
\]

(6.1) (6.2) (6.3)

where \(y_i \in \{0,1\}\) for all \(i \in M\) depicts whether a bin \(i\) is used \((y_i = 1)\) or not \((y_i = 0)\) and where \(x_{ij} \in \{0,1\}\) for all \(i \in M\) and \(j \in N\) is 1 if item \(j\) is assigned to bin \(i\). It is assumed that \(c\) is a positive value and that this value is larger than every weight \(w_j\). The constraints (6.2) ensure that the combined weight of all items which are packed into one bin do not exceed its capacity and the constraints (6.3) depict that for each item \(j\) exactly one bin \(i\) has to be assigned. The objective of this problem (6.1) is to minimize the number of used bins.

### 6.2.2 Mathematical model for the maintenance scheduling problem

The BPP is used as a base for the modelling of the maintenance scheduling of the plant in Amiens. Therefore, the maintenance tasks \(T = \{1, ..., T_{\text{max}}\}\) which have to be scheduled correspond to the items in the BPP while the potential maintenance slots \(S = \{1, ..., S_{\text{max}}\}\) are depicted as bins. The decisions to perform a maintenance task \(t \in T\) in maintenance slot \(s \in S\) is depicted by the binary variables \(x_{s,t}\) with \(x_{s,t} = 1\) if \(t\) is processed in \(s\) and the decision that the production line \(l \in L = \{1, ..., L_{\text{max}}\}\) is maintained during the maintenance slot \(s \in S\) is depicted by the binary variables \(y_{sl}\) where \(y_{sl} = 1\) indicates that the \(l\) has to be shut down due to maintenance in \(s\). Analogously to the constraints (6.3) for each maintenance task \(t\) exactly one maintenance slot \(s\) has to be chosen in which the task will be performed:

\[
\sum_{s=1}^{S_{\text{max}}} x_{s,t} = 1 \quad \forall t \in T.
\]

(6.4)

While the original problem only takes the weight of the items into account, several constraints have to be considered in order to adequately model the scheduling problem described in section 6.1. First of all, slots \(s \in S\) and tasks \(t \in T\) have different total durations depicted by the parameters \(D_{s}^{\text{slot}}\) and \(D_{t}^{\text{task}}\). Hence, a task can only be carried out in a maintenance slot which is at least as long as the task needs to be completed:

\[
x_{s,t}D_{t}^{\text{task}} \leq D_{s}^{\text{slot}} \quad \forall s \in S, t \in T.
\]

(6.5)

Certain workers with specific skills are required to fulfill a specific task. In order to not model each individual worker who works at the plant, different groups of workers \(W = \{1, ..., W_{\text{max}}\}\) (e.g. electricians, mechanics, ...) are introduce. During each maintenance slots \(s\) a certain number of person-hours per group \(W_{s,t}^{\text{slot}}\) is available to cover the workload \(W_{t}^{\text{task}}\) of a task \(t\). The number of available person-hours restricts the number of tasks that can be carried out during a maintenance slot:
Beside these capacity restrictions each maintenance task has to be performed in a given time interval

\begin{align}
 x_{s,t}(s - E_t) &\geq 0 \quad \forall s \in S, \ t \in T \label{eq:6.7} \\
 x_{s,t}(L_t - s) &\geq 0 \quad \forall s \in S, \ t \in T \label{eq:6.8}
\end{align}

where the parameter \( E_t \in S \) depicts the earliest and \( L_t \in S \) the latest timeslot in which a task \( t \) might be carried out. If \( x_{s,t} = 1 \) then the chosen slot \( s \) has to be in the interval \([E_t, L_t]\) to satisfy both equations \((6.7)\) and \((6.8)\).

In order to keep track of the slots that are used for maintenance at a production line \( l \in L \) the following constraints are introduced

\begin{align}
 x_{s,t} \leq y_{s,l} \quad \forall s \in S, \ l \in L, \ t \in \{t' \in T | P_{t'} = l\} \label{eq:6.9}
\end{align}

where \( P_t \) denotes the line which is associated with a task \( t \). Also not all lines can be shut down for maintenance at any regarded timeslot, which is modelled by the following constraints:

\begin{align}
 y_{s,l} \leq A_{s,l} \quad \forall s \in S, \ l \in L \label{eq:6.10}
\end{align}

The parameters \( A_{s,l} \) depict whether the production line \( l \) can be maintained \( (A_{s,l} = 1) \) during the slot \( s \) or not \( (A_{s,l} = 0) \).

Maintenance tasks have different priorities. If a task \( t \) with a priority \( p_t \) is carried out in slot \( s \) then no other task \( t' \) with a lower priority \( p_{t'} \) is allowed to be carried out in a maintenance slot \( s' \) with \( E_t \leq s' \leq s \). The latter restriction permits tasks with a lower priority to be performed before a task with a higher priority if the task with a higher priority cannot be carried out in this timeslot because of the constraint \((6.7)\):

\begin{align}
 (1 - x_{s,t})T_{\text{max}} \geq \sum_{t' \in \{t' \in T | P_{t'} = l\} \setminus \{t'' \mid p_{t''} < p_t\}} \sum_{s' = E_t}^{s-1} x_{s',t'} \label{eq:6.11}
\end{align}

The objective of this optimization is to minimize the number of maintenance slots needed to carry out all tasks and to delay all tasks as much as possible. Unfortunately, it is not always possible to find one solution which is optimal at the same time for both cases. For example for \( S = \{1,2\} \) and \( T = \{1,2\} \) with \( E_t = 1 \) for \( t \in T \) and \( L_1 = 1 \) and \( L_2 = 2 \) it is either possible to perform both tasks in the first slot to minimize the number of used maintenance slots or it is also possible to use both maintenance slots in order to delay the tasks as much as possible. Both options are valid solutions and a decision has to be made which solution is preferred. In case of the P&G plant it is much more important to minimize the number of needed maintenance slots to free up time for production.

Hence, the following objective function is used:

\begin{align}
 \min M \cdot \left( \sum_{i=1}^{L_{\text{max}}} \sum_{s=1}^{S_{\text{max}}} y_{s,i} \right) + \sum_{i=1}^{L_{\text{max}}} \sum_{s=1}^{S_{\text{max}}} (S_{\text{max}} - s) y_{s,i} \label{eq:6.12}
\end{align}

with \( M > L_{\text{max}} \cdot \sum_{s \in S} s \) being a factor which is used to weight the decision to use less slots much higher than any decision to use a later maintenance slot. The first part of the objective function \( \left( \sum_{i=1}^{L_{\text{max}}} \sum_{s=1}^{S_{\text{max}}} y_{s,i} \right) \) depicts the minimization of the number of slots while the second part \( \sum_{i=1}^{L_{\text{max}}} \sum_{s=1}^{S_{\text{max}}} (S_{\text{max}} - s) y_{s,i} \) models the preference of later slots over earlier slots. Due to the factor
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$M$ the first part of the objective is dominant. Hence, a solution which uses one slot more than another solution cannot lead to a better objective value than a solution with fewer slots.

6.3 Algorithmic approach for improved priority handling

Following the strict policy for the handling of priorities induced by the restrictions (6.11) can lead to solutions, which could be further improved.

![Current solution diagram](image)

**Current solution:**

<table>
<thead>
<tr>
<th>Slot</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

![Improved solution diagram](image)

**Improved solution:**

<table>
<thead>
<tr>
<th>Slot</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

*Figure 6.1: Example for a possible improvement of solutions by using sparse capacities in maintenance slots.*

Figure 6.1: Example for a possible improvement of solutions by using sparse capacities in maintenance slots. shows an example how the sparse capacities of a maintenance slot can be used for lower priority tasks without interfering with the assignment of other high priority tasks. In the depicted case it is possible to reduce the number of necessary maintenance slots by moving task 4 to slot 1. While the strict restrictions of the model would not allow such a solution in practice this might not be the case. Therefore, an algorithm (see Algorithm 1) was developed to find solutions, which utilize the sparse capacities of single slots.

![Algorithm 1 diagram](image)

**Algorithm 1: Algorithm for iterative scheduling**

The main idea of the developed algorithm is to split the set of tasks into multiple disjoint sets of tasks $T = T_1 \cup \ldots \cup T_{p_{\text{max}}}$ with $p_{\text{max}}$ being the highest priority and $T_p = \{ t \in T | p_t = p \}$ being the set of all tasks with the priority $p$. Afterwards all sets of tasks are scheduled iteratively in a descending order according to their priority. By scheduling the tasks in this order single tasks are only allowed to
use the sparse capacities if these capacities were not needed for the maintenance of tasks of all higher priorities. After each set of tasks was successfully scheduled, using the optimization model described in section 6.2.6.2 the available person-hours $W_{s,l}^{slot}$ have to be reduced and all variables $y_{s,l}$ have to be fixed to 1 if $y_{s,l} = 1$ in the optimal solution.

6.4 Results & outlook

The model and algorithm were implemented using the programming language Julia 0.6.2 [23] using the package JuMP [24] to implement the developed mathematical optimization problem. To solve the MILP problem the commercial solver IBM ILOG CPLEX 12.7.1 was used. The approach was tested using a MS Windows 7 desktop PC with an Intel Core i7-4770 CPU @ 3.40 GHz and 24 GB of RAM. We used historical data for the last 18 Months of one production line to test the MILP formulation and the iterative algorithm. The results of the optimizations in contrast to the historical data can be seen in Figure 6.2.
Using both approaches the number of maintenance slots, which are necessary to perform all tasks, was reduced to 26 from originally 59. The computation time needed to obtain these results was around five seconds. While the achieved results are much better w.r.t. the proposed objective function it cannot be neglected that, several tasks might have been scheduled at a specific timeslot due to sudden breakdowns or other criteria although it would have been beneficial to delay the task to another maintenance slot. It has to be evaluated, whether the promising results can also be achieved during the real production. Therefore, a first tool was developed which the real planners at P&G will use to evaluate the method in a real-world setting. Besides improving the tool according to the feedback provided by the end-users, the next important steps to achieve the overall goals of the work package are to integrate the improved predictions for necessary maintenance provided by data-analysis. Also an integration of both methods into the real system at the plant has to be considered.

7 Concluding remarks, current and future work

The first part of this report presents several modelling attempts for the optimization of the short-term production scheduling of multi-product, multi-purpose and multi-stage production facilities typically met in the process industries such as the food plant of FRINSA. More specifically, three modelling approaches are proposed in total. Approach A, successfully schedules a real weekly demand in the packing stage of the plant using an MILP model based on the immediate-precedence framework. The main contribution of this approach is the recognition of the sterilization stage as the production’s bottleneck. Therefore, the implementation of these results in the real plant will produce a sub-optimal solution. According to these observations, an alternative approach B is developed, which includes both the sterilization and packing stages of the FRINSA plant. A decomposition technique combined with a general precedence-based MILP model is proposed to solve this challenging problem. It was proven that the proposed approach is able to solve the problem, but it is limited to medium-scale problem instances. The deeper understanding of the process after multiple discussions with the plant operators, and the conclusions of the two first approaches, helped in the development of the last presented modelling approach. The categorization of the products into product families and the use of a decompositions technique utilizing effective insertion criteria, proved to significantly reduce the required computational effort to solve the problem. A real-week plant operation was successfully scheduled requiring small computational times. This is one of the first successful and systematic attempts to solve a mixed batch and continuous industrial scheduling problem of this size. An important aspect of approach C is that headroom still exists for further improvement of the schedule. Even more demanding weeks can be scheduled, while an extension to include all processing stages is possible.

It should be emphasized that while specific attention has been made to the modelling of food plants, such as the FRINSA use case, the proposed models are generic enough to be used in other types of process industries such as chemical, pharmaceutical, etc.
Despite the promising results of the suggested MIP-based solution strategy, work is currently in progress to address more complex and realistic issues of the underlying problem. Thus, ongoing work focuses on the following activities:

- Further improvement of the initial schedule using reallocation and reinsertion actions.
- Introduction of multiple due dates for the same product.
- Modelling the shared resources (carts) between the sterilization and packing stage.

The proposed modelling and optimization frameworks will be completed with the following future steps:

- Introduction of cost issues in the objective of the model (sterilizer’s energy cost, unit production costs, changeover costs).
- Modelling of the remaining processing stages.
- The considerations of more demanding problem instances.
- The consideration of uncertainties e.g. order cancellations/modifications with the implementation of a rolling horizon approach.

In the second part of this deliverable an optimization model for the planning of maintenance activities was proposed. The overall formulation has been inspired by the P&G use case and it is based on the Bin Packing Problem. The solutions obtained by applying this method were further improved by using a new algorithm which is utilizing unused capacities due to too strict priority handling of the maintenance tasks. Both, the mathematical model and the new algorithm were evaluated using historical data. These first tests showed promising results.

In the future, the developed methods have to be tested at the real plant in Amiens. Therefore, a prototype tool is being developed which will be handed to the planners at the plant. After an evaluation by the end-users, the developed method will be adjusted according to their feedback. Beside testing the new methods, a combination of the scheduling part and the data-analysis part of this work-package has to be done in order to help P&G moving from a time-based maintenance to a more efficient predictive maintenance.

Overall, Deliverable 4.1 fully meets the original objectives related to Task 4.1. It is worthwhile to note that it also covers activities related to Task 4.4 “validation studies” which started before the originally scheduled period.
8 References


Techniques for plant-wide reactive scheduling
Modelling frameworks, solution approaches and validation


